MAT 1348 3X – Test # 1 – Spring/Summer 2016

Full Name: Solutions

Student Number: _____

Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature: _____

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Full Name:_____

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Question	Possible Points	Points Obtained
# 1	5	
# 2	5	
# 3	5	
# 4	5	
# 5	5	
Total	25	

Instructions:

- Print your name and student number on the first two pages.
- Verify that your copy of the test has all of its 9 pages.
- You must answer all questions. There are 5 questions for a total of 25 points.
- Write the solutions to the questions in the space provided. You may use the back of the pages if necessary.
- This a closed book test, no course notes are permitted.
- Calculators are permitted.

SHOW ALL YOUR WORK

1. **(5 pts)** Use the method of your choice to determine if the following compound proposition is a tautology, a contradiction or a contingency. If you determine that the compound proposition is a contingency, give at least one truth assignment for the propositional variables for which the compound proposition will be false.

$$\underbrace{((r \to (\neg p \wedge q)) \leftrightarrow ((\neg p \to q) \to \neg r))}_{\varphi}.$$

① By Truth Table:

	p	q	r	$\neg p$	$\neg p \wedge q$	$r \to (\neg p \wedge q)$	$\neg p \to q$	$\neg r$	$(\neg p \to q) \to \neg r$	φ
	T	T	T	F	F	F	T	F	F	T
	T	T	F	F	F	T	T	T	T	T
	T	F	T	F	F	F	T	F	F	T
	T	F	F	F	F	T	T	T	T	T
a)	F	T	T	T	T	T	T	F	F	F
	F	T	F	T	T	T	T	T	T	T
b)	F	F	T	T	F	F	F	F	T	F
	F	F	F	T	F	T	F	T	T	T

② By Truth Tree:

① $\neg\varphi$ x

from ①: $r \to (\neg p \wedge q)$ x ; $(\neg p \to q) \to \neg r$ x

③ $\neg((\neg p \to q) \to \neg r)$ x ; $\neg(r \to (\neg p \wedge q))$ x

from ③: ④ $(\neg p \to q)$ x ; ⑤ r

⑥ $\neg\neg r$; $\neg(\neg p \wedge q)$ x

from ②: $\neg r$ x ; $(\neg p \wedge q)$ x ; $\neg(\neg p \to q)$ x ; $\neg r$ x

from ⑥: ⑦ $\neg p$; ⑧ q ; $\neg p$; $\neg q$

from ④: $\neg p$ a) ; $\neg q$ x

from ⑤: $\neg\neg p$ x ; $\neg q$ b)

open branches imply not a tautology. Since only two out of eight assignments make the formula false (by using open branches) it is not a contradiction.

∴ formula is a contingency with the following truth assignments making it false

	p	q	r
a)	F	T	T
b)	F	F	T

2. **(5 pts)** An inhabitant B on the island of knights and knaves is accused of a crime. His lawyer, A, is also an inhabitant of the island. During the proceedings, the following declarations are made:

A : My client is a knight only if he is guilty.

B : My lawyer is a knave if and only if I am innocent.

(a) Can you determine what the type of inhabitants are A and B? If yes, indicate which is which. If not, explain briefly why you cannot.

(b) Can you determine whether B is innocent? If yes, indicate if he is guilty or innocent. If not, explain briefly why you cannot.

Let p: A is a knight ~~and~~ q: B is a knight, and r: B is innocent.

$\therefore$ A: $q \rightarrow \neg r$

B: $\neg p \leftrightarrow r$

① By Truth Table:

p	q	r	$\neg r$	$\neg p$	$q \rightarrow \neg r$	$\neg p \leftrightarrow r$	possible?
T	T	T	F	F	F	F	X
T	T	F	T	F	T	T	✓
T	F	T	F	F	T	F	✓
T	F	F	T	F	T	T	X
F	T	T	F	T	F	T	✓
F	T	F	T	T	T	F	X
F	F	T	F	T	T	T	X
F	F	F	T	T	T	F	X

② By Reasoning (cases):

i) Suppose A is a knight, then p is T and $q \rightarrow \neg r$ must be T. Hence either q is F or $\neg r$ is T.

- if p is T and q is F, then B is a knave which makes $\neg p \leftrightarrow r$ F. Thus, p is T, q is F and r must be T.
- if p is T and $\neg r$ is T, then $\neg p \leftrightarrow r$ is T. This implies B is a knight so q is T. Thus, p is T, q is T and r is F

Extra page.

ii) Suppose A is a knave, then p is F and $q \to \neg r$ must be F. Hence q is T and $\neg r$ is F. If q is T then B is a knight which requires $\neg p \leftrightarrow r$ to be T, which is the case since $\neg p$ is T and r is T.

Conclusions:

a) We cannot determine the type for A nor B since there are 3 cases which can arise and they can be of either type among them.

b) We cannot determine the innocence of A since within the 3 possible cases he can be innocent or guilty.

3. **(5 pts)** Translate the following argument into argument form. Use the following propositional variables:

   b: The dog barks.

   r: Someone rings the doorbell.

   h: The dog is at home.

   g: Guy opens the front door.

   s: The dog sleeps.

   "If the dog barks, then the dog is not at home unless it is sleeping. If the dog is at home, then someone rings the doorbell only if the dog barks. For the dog to bark, it is sufficient that Guy opens the front door. Therefore, someone rings the doorbell if and only if the dog barks."

DO NOT VERIFY THE VALIDITY OF THIS ARGUMENT

$H1: \quad b \rightarrow (\neg s \rightarrow \neg h)$ ← also $\neg(s \vee \neg h)$

$H2: \quad h \rightarrow (r \rightarrow b)$

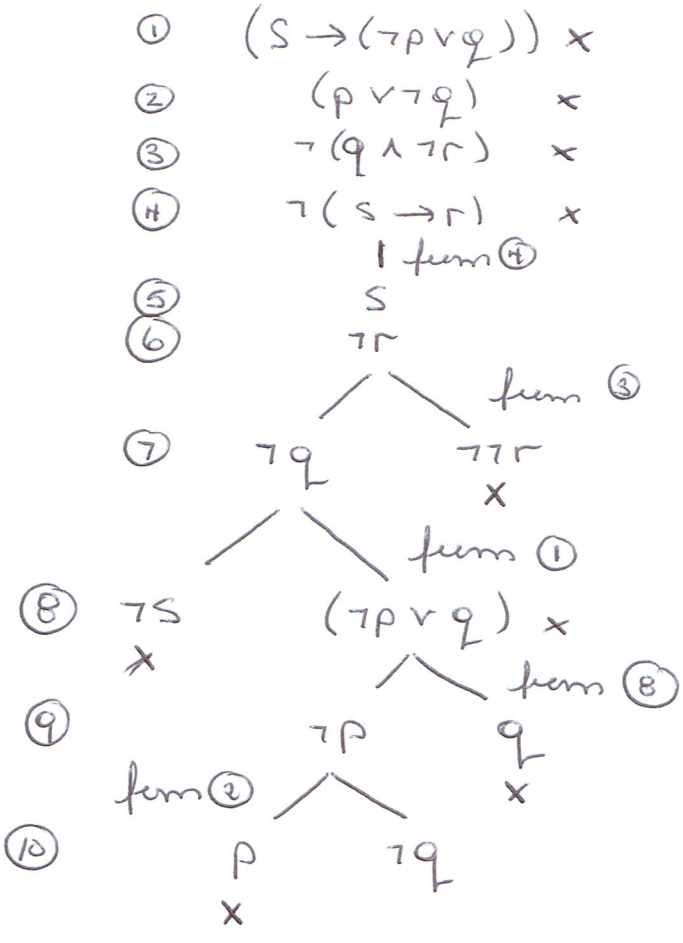
$H3: \quad g \rightarrow b$

$\therefore C: \quad r \leftrightarrow b$

4. **(5 pts)** Verify the validity of the following argument. If the argument is invalid, give a counter-example.

$$\frac{\begin{array}{c}(s \to (\neg p \vee q)) \\ (p \vee \neg q) \\ \neg(q \wedge \neg r)\end{array}}{(s \to r)}$$

① By Truth Tree:



The argument is invalid, since there are open branches.

counter-examples:

p	q	r	s
F	F	F	T

5. **(5 pts)** Use the method of your choice to find a proposition in DNF equivalent to the following compound proposition:

$$(\neg(r \to p) \leftrightarrow (p \wedge q))$$

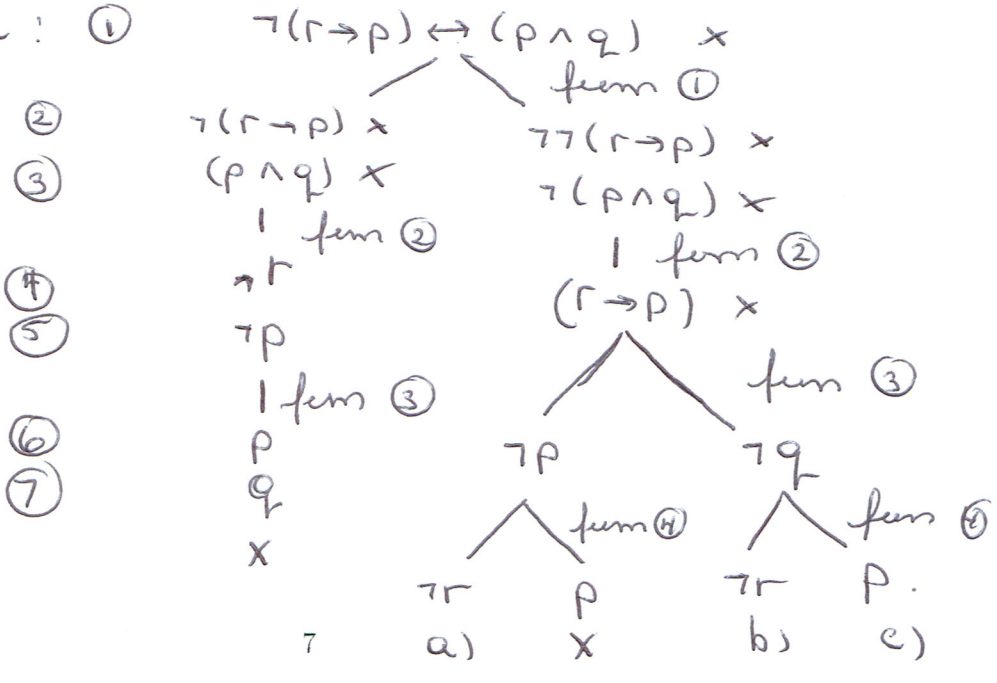
① By Truth Table:

p	q	r	$r \to p$	$\neg(r \to p)$	$p \wedge q$	$\neg(r \to p) \leftrightarrow (p \wedge q)$	
T	T	T	T	F	T	F	
T	T	F	T	F	T	F	
T	F	T	T	F	F	T	✓ a)
T	F	F	T	F	F	T	✓ b)
F	T	T	F	T	F	F	
F	T	F	T	F	F	T	✓ c)
F	F	T	F	T	F	F	
F	F	F	T	F	F	T	✓ d)

$\therefore (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$

b) a) c) d)

② By Truth Tree: ①



$(\neg p \wedge \neg r) \vee (\neg q \wedge \neg r) \vee (p \wedge \neg q)$

a) b) c)

③ By Alg manipulation.

$$\neg((r \to p) \leftrightarrow (p \wedge q))$$

$$\equiv (\neg(r \to p) \wedge (p \wedge q)) \vee (\neg\neg(r \to p) \wedge \neg(p \wedge q))$$

$$\equiv (((r \wedge \neg p) \wedge (p \wedge q)) \vee ((\neg r \vee p) \wedge (\neg p \vee \neg q))$$

$$\equiv F \vee (((\neg r \wedge \neg p) \vee (\neg r \wedge \neg q) \vee (p \wedge \neg p) \vee (p \wedge \neg q))$$

$$\equiv (\neg p \wedge \neg r) \vee (\neg q \wedge \neg r) \vee (p \wedge \neg q)$$